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SHORT NOTE

A Mohr circle construction to plot the stretch history of material lines

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Abstract—Mohr circles for stretch can be used to find the spatial distribution of material lines which have been shortened, extended, shortened-then-extended and extended-then-shortened in a material plane which underwent area-constant homogeneous deformation by steady-state flow. The distribution is mainly a function of the vorticity of the parent flow, and can be used to determine sense of shear, finite strain, and even vorticity and orientation of finite strain axes. The method may be useful in theoretical and experimental studies and can help to understand the complex deformation patterns in sets of cross-cutting veins in deformed rocks.

INTRODUCTION

IN MANY deformed terrains, sets of cross-cutting veins can be found which are boudinaged, folded, or folded and boudinaged in sequence. If the veins are all pre-kinematic, the specific type of deformation will depend on the orientation of the vein. Talbot (1970, 1987), Hutton (1982) and Passchier (1986) have demonstrated that such sets of veins provide important data from which at least finite strain and sense of shear can be determined. This is done using the geometric distribution of boundaries between sets of veins with a similar stretch history, i.e. without actually determining the stretch along the veins. In theory it is even possible to obtain data on the actual flow vorticity and on area change in the plane of observation. Similar techniques could be applied in general to any set of deformed material lines. However, calculation of the possible arrangements of the boundaries is a relatively cumbersome procedure. In this paper, a simple construction method is presented which uses Mohr circles for stretch to predict the geometry of sectors of material lines with a similar stretch history on any plane in a deformed material. The construction method only applies to constant-area deformation produced by steady state flow and therefore mainly serves to illustrate the influence of vorticity and finite strain on sets of differently orientated material lines or veins. The method can, however, be expanded to involve area change and other parameters associated with natural deformation (Passchier in preparation, Passchier & Talbot in preparation).

MOHR CIRCLES FOR STRETCH

Homogeneous finite deformation in a plane (Fig. 1c) is fully described by the equations;

$$x' = \mathbf{F}\cdot x \text{ and } x = \mathbf{H}\cdot x',$$

where x and x' are the Cartesian co-ordinates of material points in the undeformed and deformed state, respectively. **F** is the Lagrangian (material) tensor relating particle positions in the undeformed state to their positions in the deformed state. **H** is the Eulerian (spatial) tensor relating particle positions in the deformed state to their positions in the undeformed state (Fig. 1c). De Paor (1981) and Means (1982) introduced Mohr circles which can be used as a graphical expression of these tensors, as shown in Fig. 2(b) for **F**. Polar co-ordinates of each point on the circle represent the rotation and

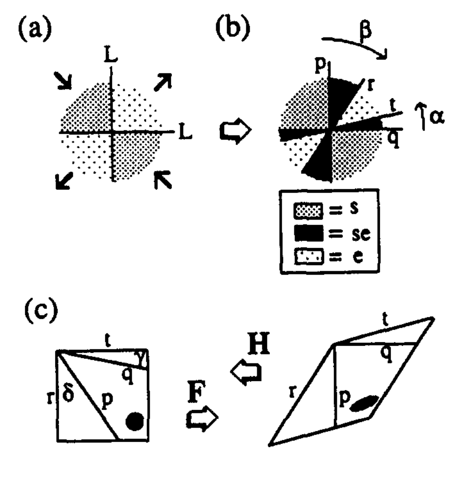

Fig. 1. (a) Distribution of sectors of shortening (dark) and extending (light) material lines in homogeneous constant-area flow. Short arrows indicate axes of maximum and minimum instantaneous stretching rate. L-axes are lines of no instantaneous stretching rate. This is the 'eigen'-flow type which would lead to the finite deformation in (b) after some time. Material line pairs r,t and p,q coincided with the L-axes at the start and the end of the deformation period, respectively. α and β are the angles over which t and r rotated. p,q,r and t separate sectors where material lines extended (e), shortened (s) or first shortened, then extended (se); (c) deformation of the set of material lines (p,q,r,t) as described by tensors **F** and **H**.

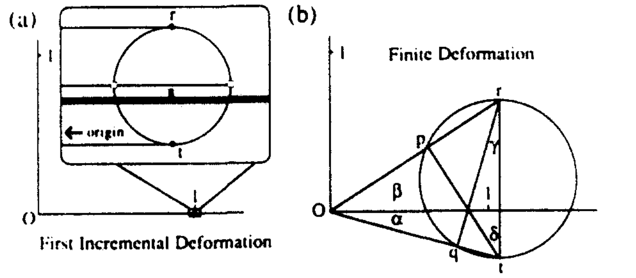

Fig. 2. (a) F-Mohr circle (enlarged) for the first step of incremental deformation in response to the 'eigen'-flow type of Fig. 1(a); material lines coinciding with the axes of maximum and minimum stretching rate (squares) plot on a horizontal line through the circle center. Lines r and t plot normal to these lines in the Mohr circle; (b) F-Mohr circle for the finite deformation in Figs. 1(b) & (c). Construction method to find p,q,r,t explained in text.

stretch of a material line in response to the deformation (Means 1982); for example, β and distance (Op) in Fig. 2(b) represent the rotation and stretch of the material line p in Fig. 1(c). The angle between material lines, as measured around the circle for **F**, is twice the angle measured in 'real' space in the deformed state.

'EIGEN'-FLOW TYPE

The tensors **F** and **H** describe finite deformation, i.e. the displacement of material points or lines between an original and final configuration. As such, they do not contain any information on deformation history; a finite deformation state can be formed by an unlimited number of flow (or instantaneous deformation) regimes and deformation paths. Nevertheless, each finite deformation state can be linked to one unique flow type by which it would form in the case of steady state flow, i.e. if flow parameters did not change in magnitude in the course of progressive deformation. This can be defined as the 'eigen'-flow type of that finite deformation state. Although such a flow type did not necessarily contribute to the finite deformation state, it is nevertheless useful as a standard setting, against which more realistic situations can be assessed.

FLOW AND FINITE DEFORMATION

If surface area is preserved, as for the flow type shown in Fig. 1(a), axes of maximum and minimum stretching rate (arrows) lie at 45° to orthogonal axes of no infinitesimal longitudinal strain (L-axes). L-axes are spatial lines which separate sectors of instantaneously shortening (dark) and instantaneously extending (light) material lines. During progressive deformation, material lines will continually 'pass' the L-axes of flow from the instantaneous shortening to the instantaneous extension sector (Fig. 1a), resulting in two sectors where material lines have first been shortened, then extended (*se*; Fig. 1b) or first extended, then shorted (*es*). Two additional sectors will develop where material lines have been shortened (*s*) or extended (*e*) only (Fig. 1b). The presence of *se* or *es* sectors depends on flow vorticity, as explained below. In Fig. 1(b), the four sectors in the finite deformation state are separated by four material lines, which coincided with L-axes of flow at the onset of deformation (r and t), or at the very end of the deformation period (p and q).

CONSTRUCTION METHOD

It is possible to use a simple geometric construction method in a Mohr circle for **F** or **H** to find the distribution of the *s*, *e*, *se* and *es* sectors of material lines in any finite deformation state resulting from steady state 'eigen'-flow fixed in an external reference frame. The discussion is restricted to a circle for **F**, but construction in a circle for **H** is analogous.

The angle measured around the Mohr circle for **F** equals the angle between material lines in the undeformed state (Means 1982). For the first incremental deformation step, the angle over which these material lines rotated was infinitesimally small. Now consider an **F**-Mohr circle for this first deformation increment (Fig. 2a); material lines which coincided with axes of maximum and minimum stretching rate at the onset of deformation will have undergone maximum and minimum stretch values, i.e. they plot on the Mohr circle nearest to and furthest from the origin of the reference frame. Since rotations are infinitesimal, the lines lie on a horizontal line through the center of the Mohr circle (Fig. 2a; squares). The material lines which coincided with L-axes for the first incremental deformation step (r and t) lie at 45° to axes of maximum and minimum stretching rate in real space, so they plot on a vertical axis through the circle center. These points will lie in the same positions on **F**-Mohr circles for finite deformation (Fig. 2b) (Means 1982, Passchier 1988b). It is now easy to prove on geometric grounds that material lines p and q in Fig. 1 coincide with the intersection of the Mohr circle and the tie lines from r and t to the origin of the diagram in Fig. 2(b). α and β are the angles over which q and t, and p and r, respectively, rotated in the external reference frame due to finite deformation. In a Mohr circle for **F**, γ and δ are, respectively, the angles between t and q, and between r and p in the undeformed state (Means 1982). Since r–p–t and t–q–r are right angles, $\delta = \beta$ and $\gamma = \alpha$. This means that p and q rotate exactly in such a way, that in the deformed state they occupy the positions which r and t occupied in the undeformed state (Fig. 1c). Points p, q, r and t in Fig. 2(b) therefore correctly represent the material lines p, q, r and t in Figs. 1(b) & (c), which separate sectors of material lines with similar stretch history.

THE EFFECT OF VORTICITY

Passchier (1988b) has shown that in the Mohr circle for stretch, the elevation of the circle center above the horizontal axis, divided by the circle radius is a measure for the vorticity of the 'eigen'-flow for that finite deformation state. This ratio is known as the kinematic vorticity number *Wk* (Means *et al.* 1980). $Wk = 0$ corresponds to pure shear (coaxial) flow; $Wk = 1$ to simple shear flow, $Wk > 1$ to a rotational simple shear (Passchier 1986, 1988a,b). In Fig. 3, four finite deformation situations are illustrated by F-Mohr circles for different vorticity numbers of 'eigen'-flow. Using the construction described above to find the distribution of material line sectors, it is clear that *Wk* has a profound influence on the geometry of this distribution. The following conclusions can now be drawn;

(1) for pure shear progressive deformation ($Wk = 0$) *se* sectors are always of equal size;

(2) for simple shear progressive deformation ($Wk = 1$), only one *se* sector exists. The line separating *s* and *e* sectors is the flow plane of simple shear;

(3) if $0 < Wk < 1$, the resulting deformation has two *se* sectors of different size. Their arrangement is dependent on sense of shear;

(4) if $Wk > 1$, one *se* and one *es* sector exist;

(5) the relative size of *e*, *s*, *se* and *es* sectors depends on the vorticity of the characteristic flow, while absolute magnitudes are a function of finite strain as well.

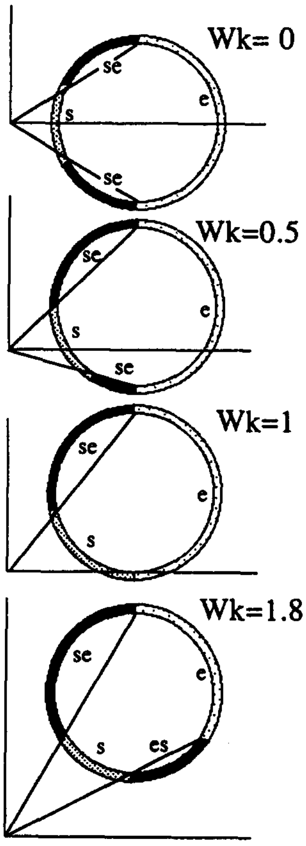

Fig. 3. F-Mohr circles for four different vorticity numbers of 'eigen'-flow. Tie lines have been drawn to find distribution of *e*, *s*, *se* and *es* sectors in each case. Explanation in text.

APPLICATION

The construction method described here can be useful in a wide range of situations. It can be used in teaching. It is directly applicable to 'two-dimensional' deformation experiments in shear boxes and in transparent deformation cells such as those used to study rock analogues; and it may be used to calculate deformation and 'eigen'-flow parameters from the orientation of material line sectors. If the lines *p*, *q*, *r* and *t* can be found in a deformed material, the Mohr circle for **H** can be constructed and deformation parameters such as R_f, *Wk* of the 'eigen'-flow type, and the orientation of finite strain axes can be read from the circle. This may eventually even be applied to vein sets in naturally deformed rocks, provided that a correction can be made for area change, and for the fact that sectors of folded and boudinaged veins will not coincide precisely with *s* and *e* sectors due to layer-parallel shortening and extension (Passchier in preparation, Passchier & Talbot in preparation).

REFERENCES

De Paor, D. G. 1981. Geological strain analysis. Unpublished Ph.D. thesis, National University of Ireland.

Hutton, D. H. W. 1982. A tectonic model for the emplacement of the main Donegal granite, NW Ireland. *J. geol. Soc. Lond.* **139**, 615–631.

Means, W. D. 1982. An unfamiliar Mohr circle construction for finite strain. *Tectonophysics* **80**, T1–T6.

Means, W. D., Hobbs, B. E., Lister, G. S. & Williams, P. F. 1980. Vorticity and non-coaxiality in progressive deformation. *J. Struct. Geol.* **2**, 371–378.

Passchier, C. W. 1986. Flow in natural shear zones—the consequences of spinning flow regimes. *Earth Planet. Sci. Lett.* **77**, 70–80.

Passchier, C. W. 1988a. Analysis of deformation paths in shear zones. *Geol. Rdsch.* **77**, 309–318.

Passchier, C. W. 1988b. The use of Mohr circles to describe non-coaxial progressive deformation *Tectonophysics* **149**, 323–338.

Talbot, C. J. 1970. The minimum strain ellipsoid using deformed quartz veins. *Tectonophysics* **9**, 46–76.

Talbot, C. J. 1987. Strains and vorticity beneath a tabular batholith in the Zambesi belt, northeast Zimbabwe. *Tectonophysics* **138**, 121–158.